in Eq. (7) of Ref. 4:

$$\Delta/a_0^2\rho_0 \leq (1/X + 1/A)^{-1} . \tag{9}$$

Since $A = \frac{1}{3}\pi^2(T^2/\Theta_D'^2) \ll 1$, the bound (9) is much smaller than (8). When viewed as a function of X the right-hand side of (9) increases monotonically with increasing X, the maximum (asymptotic) value being $A(\ll 1)$, whereas (8) peaks at $X \sim 1$ with a peak value $\gtrsim \frac{1}{4}$. The expressions (8) and (9) are approximately equal only for $X \gg 1/A$ and $X \ll A$.

The reason for the difference between the results (8) and (9) lies solely in the choice of trial function. The result (9) was obtained[4] with a trial function, which was a variable linear combination of the solution of the impurity-only problem with the solution of the phonon-only problem. The trial function of RB (2.15), however, contains no variational parameter; it is an indication of how unfortunate this ansatz is, that the simplest conceivable trial function, namely, an energy-independent constant [say $\tau(\epsilon) = \tau_{ep}^0$] would do far better than the ansatz of RB (2.15). Such a constant trial function gives immediately

$$\Delta/a_0^2\rho_0 \leq A , \tag{10}$$

which eliminates the peak exhibited by (8) when plotted as a function of X.

The connection with our discussion prior to Eq. (6) may now be made. The bound (9) is simply a combination in *parallel* of the resistivities Δ_1 and Δ_2 as defined by (3′) and (5), since $\Delta_1 = a_0^2\rho_{ep}^0$ and $\Delta_2 = a_0^2 A\rho_0$. Then (9) assumes the form

$$\Delta \leq (1/\Delta_1 + 1/\Delta_2)^{-1} . \tag{11}$$

Clearly (11) is in accordance with the requirement that Δ should be less than or equal to the smaller of Δ_1 and Δ_2. It is also apparent that the relative magnitude of ρ_1 and ρ_2 is of no immediate relevance.

With the obvious identifications the result (8) due to RB is similarly written as

$$\Delta = \Delta_1[\rho_2^2/(\rho_1+\rho_2)^2] + \Delta_2[\rho_1^2/(\rho_1+\rho_2)^2] , \tag{12}$$

which has the form of a *series* combination of the resistivities Δ_1 and Δ_2 after multiplication with weight factors involving the ratio ρ_1/ρ_2. From (12) one may recover the form (11) only by the replacement $\rho_1 \rightarrow \Delta_1$ and $\rho_2 \rightarrow \Delta_2$ in the weight factors. Once again, this demonstrates the relevance of the ratio Δ_1/Δ_2 rather than ρ_1/ρ_2.

The plots shown in RB's Fig. 1 were obtained by neglecting the second term in (8) compared to the first one, that is, by writing

$$\Delta = a_0^2\rho_0 \frac{X}{(1+X)^2} = a_0^2 \frac{\rho_{ep}^0}{(1+\rho_{ep}^0/\rho_0)^2} . \tag{13}$$

This approximation corresponds to setting $A = 0$ in (9), which results in $\Delta = 0$. Therefore (13) should be interpreted as nothing more than an upper bound on the number zero.

We conclude that Refs. 1–3 seriously overestimate the deviations from additivity of the electron-phonon and electron-impurity scattering, and that the predicted peaks in $\Delta(T)$ (RB's Fig. 1) do not reflect any property of the model considered, but only the inadequacy of the chosen trial function.

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*See Ref. 1.

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[1] M. J. Rice and O. Bunce, Phys. Rev. B 2, 3833 (1970), to be referred to as RB.

[2] M. J. Rice, Phys. Rev. Letters 23, 1108 (1969).

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[4] H. Smith, Solid State Commun. 8, 1991 (1970).

[5] See, e. g., J. M. Ziman, *Electrons and Phonons* (Oxford U.P., Oxford, England, 1960), Chap. VII.

[6] When the steps leading from RB's (3.2) to (3.7) are reproduced, it appears that the factors of $(1+\rho_{ep}^0/\rho_0)$ occurring explicitly in RB's (3.7) and (3.8) should not be there.

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Existence of a Gap in the Electronic Density of States of a Disordered System*

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A recent calculation by Weaire is shown to be a special case of an older and more general theory.

In a recent letter,[1] Weaire investigated the energy density of states of a particular model of an amorphous group-IV semiconductor. The Hamiltonian used was based on the tight-binding approximation to the true Hamiltonian of an electron in a disordered tetrahedrally coordinated network of atomic potentials. It is the purpose of this note to point out that Weaire's calculation is merely a particular example of a more general result[2] proved some years ago.

In Ref. 2., an electron was considered to be moving in a disordered array of identical nonoverlapping potential wells and to obey the Schrödinger equation

$$(-\nabla^2 + V - \epsilon)\psi = 0 \,, \tag{1}$$

where ψ is a real wave function. The system was then divided into cells, a cell being defined as the region closer to one particular potential well than to any other. Since ψ is continuous at each interface between cells, it follows that with periodic boundary conditions over some large volume we have

$$\sum_{\text{cells}} \int \nabla(\psi^2)\cdot d\vec{S} = 0 \,, \tag{2}$$

where $d\vec{S}$ is an element of surface area of a cell. It was shown that a gap must exist in the density of states if no cell could be found such that the integral over the cell surface was not positive. If one replaces the Schrödinger Hamiltonian of Eq. (1) by the "lattice gas" Hamiltonian of Ref. 1, then Eq. (2) reduces, in Weaire's notation, to

$$\sum_{\text{cells}} \sum_{i} (v_i^2 - u_i^2) = 0 \,,$$

from which that author derived his inequality (9).

It is worth noting that the more general theory is capable of predicting the existence of several different band gaps for suitable potentials, and that it is not necessary that the environment of any atom possess symmetry of any kind.

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Low-Temperature Thermal Conductivity of n-Ge

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The measurements of the lattice thermal conductivity of n-Ge from 0.3 to 4.2 °K by Bird and Pearlman are explained satisfactorily by the nonlinear theory of heat transfer in solids given by Kazakov and Nagaev.

I. INTRODUCTION

The lattice thermal conductivity of nonmetals of finite dimensions at low temperatures has drawn considerable attention recently. At low temperatures, the dominant scattering mechanisms are boundary scattering and impurity scattering. The anharmonic interactions (i.e., phonon-phonon interactions) may be neglected at fairly low temperatures. The first attempt to rigorously calculate the phonon conductivity of a finite lattice with defects was undertaken by Erdös.[1] Later, Kazakov and Nagaev (KN)[2] calculated nonlinear heat transfer in a lattice containing impurities using boundary conditions consistent with the experimental situation. It is normally assumed that the system departs slightly from thermodynamical equilibrium which leads to the introduction of local temperature and the linearization of the kinetic equations with respect to the temperature gradient. The KN calculations of the thermal conductivity [see Eqs. (5) and (6)] clearly show that one can calculate the effect of impurity scattering on the lattice thermal conductivity more rigorously than was done with Callaway's phenomenological approach.[3] Callaway's approach is complicated due to the fact that at one temperature there are simultaneously different scattering processes. Recent calculations[4-6] of the thermal conductivity of n-Ge show that the phonon-phonon interaction simply modifies the conductivity integral and itself gives negligible contribution for temperatures below 5 °K. These calculations suggest the use of KN's results to explain the thermal-conductivity measurements of n-Ge between 0.5 and 4.2 °K made by Bird and Pearlman (BP).

KN have considered only the isotopic scattering of the Rayleigh type for impurity scattering processes. However, in certain systems it is observed that resonance phenomenon occurs in the impurity scattering processes. To consider the effect of resonant scattering of phonons at reso-